New Criteria for Mathematical and Stochastic Modeling

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ABSTRACT- Modeling deals with the measurement of variables. A fundamental problem in almost all fields of science and especially in Mathematical and Statistical Sciences is the gap between the development of new ideas and their implementation in practice. The motivation of the Modeling is to bridge the gap between theory and practice. In the present study, the problems relating to the modeling have been described by presenting the various advanced tools for Mathematical and Stochastic modeling by considering different aspects of formulation or specification; obtaining solutions or estimating the parameters; validity or goodness of the specification; and using the models for making policy decisions. Further, a criterion for selecting non-nested linear regression model has been developed by using two stages least squares estimation.

I. INTRODUCTION:

Scientists and Engineers depend hearily on concepts of Mathematical and Stochastic Modeling. Modeling deals with the measurement of variables. Modeling involves imagination and skill. Model building or modeling is the heart of almost all fields of science. A fundamental problem in almost all fields of science especially in Mathematical and Statistical Sciences is the gap between the development of new ideas and their implementation in practice. The motivation of the modeling is to bridge the gap between theory and practice. It is the key component in Mathematical and Statistical analysis.

In recent years Mathematicians and Statisticians have shown an increasing interest in the problems of Modeling or model building. In almost all fields of science, a great deal of research has been directed to either the mathematical modeling or the stochastic modeling and establishing the functional relationships among different characteristics. Generally, these functional relationships can be expressed in terms of mathematical equations.

A set of mathematical equations concerns with two or more variables refers to a mathematical model. By introducing an error random variable or a disturbance term, the mathematical model becomes a stochastic model or statistical model. Now-a-days modeling is a new and fertile area of research in Mathematical and Statistical Sciences.

II. TYPES OF MODELING:

In the literature, broadly speaking that there are mainly two types of modeling namely (a) Mathematical Modeling and (b) Stochastic Modeling.

(a) Mathematical Modeling :

Mathematical Modeling is an abstract, simplified, mathematical construct related to a part of reality, and created for a particular purpose. The aim of mathematical modeling is to map relationships between entities in the real world to relationships between mathematical abstractions with a view of drawing conclusion about the nature of these relationships. Strictly speaking, Mathematical modeling is not a Mathematical discipline but nontheless it is an essential step in the successful application of mathematical insights to our work and our life.

Mathematical modeling essential consists of translating real world problems into mathematical problems, solving the mathematical problems and interpreting these solutions in the language of the real world.

The traditional mathematical model building proceeds along the following lines:

(i) formulation or the selection of mathematical problem to the real world problem;
(ii) deriving or obtaining situations for the selected mathematical problem; and
(iii) examing the validity or goodness of formulated mathematical problem.

(b) Stochastic Modeling:

Stochastic modeling is the art and science of using statistical techniques for the measurement of relationships between the variables. The formulation or specification of a stochastic model is an art, just as using knowledge or architecture to design a building is an art. In the specification the stochastic model, the most important variables are selected while the nonessential variables are discarded or omitted. The crucial relationships are

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formulated and incorporated in model. Best stochastic models, like best architectural designs, can serve as prototype to be followed in the future investigations. The art of specification of the best stochastic model is difficult to learn.

The traditional stochastic modeling proceeds along the following five steps:

(i) specification or formulation of the stochastic model;
(ii) estimation of the parameters of the specified stochastic model;
(iii) testing of hypotheses and constructing the confidence intervals concerning the parameters of the stochastic model;
(iv) forecasting or prediction validity of the estimated stochastic model; and
(v) using the estimated stochastic model for control and making policy decisions.

Generally the mathematical model or the stochastic model may be either in the form of a set of linear equations (linear models) or in the form of a set of nonlinear equations (nonlinear models).

III. MODEL BUILDING:

In practice model building may be carried out under three important categories.

1. Discipline Oriented Model Building:

Under discipline oriented model building, the various models are applied to analyse the research problems in one discipline only.

(a) Discipline Oriented Mathematical Model Building:

Mathematical Biology, Mathematical Economics, Mathematical Ecology, Mathematical Physics, Mathematical Ergonomy, Mathematical Psychology etc.,

(b) Discipline Oriented Stochastic Model Building:

Biometrics, Econometrics, Technometrics, Psychometrics, Criminometrics, Bioinformatics, Industrial Statistics, Business Statistics, Agricultural Statistics, Medico Statistics, Statistical Physics, Ergonometric, Psephology etc.,

2. Technical Oriented Model Building:

Under technical oriented model building, the various mathematical or stochastic models are considered for different disciplines but the choice is restricted to those models which can be understood through the particular class of mathematical or stochastic methods.

(a) Technical oriented Mathematical Model Building:

Linear Algebra, Difference and Differential Equations, Graph Theory, Calculus of variations; Linear, Nonlinear and Dynamic Programming etc.

(b) Technical Oriented Stochastic Model Building:

Descriptive Statistics, Analytical Statistics, Inductive Statistics, Inferential Statistics etc.,

3. Situation Oriented Model Building:

This concerns with the development of various Mathematical and Statistical models for certain special situations.

(a) Situation Oriented Mathematical Model Building:

Mathematical Models for Prediction, Optimization and Control, Mathematical models for some case studies.

(b) Situation Oriented Stochastic Model Building:

Operations Research Techniques, Multivariate Statistical Tools; Data Mining Techniques; Diagnostic methods etc.,

IV. CLASSIFICATION OF MATHEMATICAL AND STOCHASTIC MODELS:

This various models in the literature can be broadly classified into four types.

(i) Deterministic or Mathematical Models:
Models that are describing exact functional relationships between the variables but not involving stochastic variables.

(ii) Stochastic or Statistical Models or Probabilistic Models:
These models may contain one or more random variables.

(iii) Static Models:
In these models, the variables are independent of time.

(iv) Dynamic Models:
In these models, the variables involved are depending on time.

Some important mathematical and stochastic models discussed in the literature are:

A. MATHEMATICAL MODELS:

(i) Mathematical Models for Engineering, Management and Information Sciences.
(ii) Mathematical Models for Quantum Physics, Fluid Dynamics, Electricity and Diffusion.
(iii) Mathematical Models for Agriculture, Fisheries, Forests, Ecology, Genetics, Biology and Population Dynamics.
(iv) Mathematical Models for Medicine, Pharmacology and Life Sciences.
(v) Mathematical Models for Economics, Control and Optimization.
(vi) Mathematical Models for Space Technology.
(vii) Mathematical Epidemic Models.
(viii) Mathematical Growth Models.
(ix) Mathematical Traffic Flow Models.

(x) Mathematical Vibration Models.
(xi) Mathematical Data Mining Models and so on.

B. STOCHASTIC MODELS:

(i) Single Equation Linear Regression Model.
(ii) Simultaneous Equations Linear Regression Model.
(iii) Nonlinear Regression Models.
(iv) Partially Linear Regression Models.
(v) Multivariate Linear Regression Models.
(vi) Generalized Linear Regression Models.
(vii) Multivariate Nonlinear Regression Models.
(viii) Qualitative or Limited Dependent Variables Models: Probit, Logit and Tobit Models.
(ix) Nested and Non-Nested Regression Models.
(x) Sets of Linear Regression Models or Seemingly Unrelated Regression Equations (SURE) Models.
(xi) Random Coefficients Regression (RCR) Models.
(xii) Time Series Models: Autoregressive (AR), Moving Averages (MA), ARMA, AR Integrated MA (ARIMA), AR Conditional Heteroscedasticity (ARCH), Vector AR (VAR), VARMA, VARIMA Models.
(xiii) Stochastic Models for Forecasting.
(xiv) Econometric Models.
(xv) Operations Research Models.
(xvi) Distributed Lag Models.
(xvii) Stochastic Models for Pooling Time Series and Cross Section Data.
(xviii) Stochastic Data Mining Models and so on.

V. ADVANCED TOOLS FOR MODELING:

A wide number of techniques have been successfully developed for mathematical and stochastic modeling and applied to different situations in the real world relating to several research problems in the fields of Science and Technology. However, still there are an equally large number of situations which have not yet been modeled, because the situations may be complex or models formed are mathematically or statistically intractable.

The various tools for modeling have been studied by several American and British Mathematicians and Statisticians. In India, not much is carried out in this regard.

The main contributions made in the field of Modeling by:

Mathematical Modeling:

Bridgman (1931), Langhaar (1951), Maki and Thompson (1973), Roberts (1976), Andrews and Mc Lone (1976), Haberman (1977), Bender (1978), Kapur (1985, 1988), Cha, Rosenberg, Dym (2000), Dym (2006) and others.

Stochastic Modeling:

Rao (1965), Pesaran (1974), Hausman (1978), Amemiya (1980), Draper and Smith (1981), Davidson and Mac Kinnon (1981), Weisberg (1985), Linhart and Zucchini (1986), Bates and Watts (1988), McCullagh and Nelder (1989), Seber and Wild (1989), Wang and Chow (1994), Shi and Tsai (2002), Rencher and Schaalje (2008) and others.

(a) Tools for Mathematical Modeling:

Mathematical Modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The mathematical principles include: dimensional homogeneity and consistency, abstraction and scaling, conservation and balance principles, and consequences of linearity.

Some important tools for mathematical modeling are given by:

(i) Dimensional Analysis Method.
(ii) Geometric Scaling Method.
(iii) Approximating and Validating Methods.
(iv) Method of Optimization by Differentiation.
(v) Graphical Methods.
(vi) Monte Carlo Simulation Methods.
(vii) Analytical Methods using Differential Equations.
(viii) Numerical Methods.
(ix) Differential Difference Equations Methods.
(x) Local stability Theory and Global Methods.

(b) Tools for Stochastic Modeling:

The three main problems of stochastic modeling are

1. Selection of the stochastic Model;
2. Mis-Specification of the stochastic Model; and
3. Selection of Regressors or Variable Selection for the stochastic Model.

1. Important Criteria for Selection of the Stochastic Model:

(i) Model selection criterion using F-test
(ii) Stepwise regression techniques: Forward and Backwa selection methods.
(iii) R^2 and $\bar{R}^2$ Criteria for Model Selection.
(iv) Mallow's C_p-Criterion
(v) Amemiya's Criterion for Model Selection.

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(vi) Model Selection Criterion based on Stopping Rules using MSE Prediction.
(vii) Ullah's Criterion for Model Selection.
(viii) Omitted Variables Criterion.
(ix) Wu t-test Criterion.
(x) Akaike's Information Criterion.
(xi) Cross-Validation Method for Linear Model Selection.
(xii) Model Selection Criterion using Chow Test for Structural Change.
(xiii) Model Selection Criterion using Chow Forecast Test.
(xiv) Fisher's Information Criterion.
(xv) Cox Modified Likelihood Ratio Criterion.
(xvi) J and JA tests for Model Selection.

2. Important Mis-Specification Tests for the Stochastic Model:

(i) Ramsey's Regression Specification Error Test (RESET)
(ii) Chow test for Mis-Specification of the Stochastic Model.
(iii) Modified RESET.
(iv) The Rank Specification Error Test (RASET)
(v) The Kolomogorov Specification Error Test (KOMSET)
(vi) The Bartlett's M-Specification Error Test (BAMSET)
(vii) The White Mis-Specification Test.
(viii) Modified Likelihood Ratio, Wald and Langrange Multiplier Test.
(ix) Hausman's Test for Mis-Specification of the Stochastic Model.
(x) Information Matrix Test for Mis-Specification of the Stochastic Model.

3. Important Criteria for Variable Selection:

(i) R^2 and $\bar{R}^2$ Criteria for Selection of Regressors.
(ii) Unconditional Mean Square Prediction Error Criterion.
(iii) C_p Criterion.
(iv) Sawa's BIC Criterion.
(v) Stepwise Regression Criterion.
(vi). Predicted Error Sum Squares (PRESS) Criterion.
(vii).Multivariate Statistical Analysis such as Discriminant, Principal Components and Factor Analyses.
(viii) Posterior Odds Ratio Criterion.
(ix). Selection of Regressor using Influence Measures.
(x). Stein Rule for Variable Selection.

VI. A CRITERION FOR SELECTING NON-NESTED LINEAR STATISTICAL MODELS

The problem of selecting the best linear regression model becomes very interesting to the applied statisticians. Suppose that a statistician has several possible competing models, from which he has to choose the best model (or) to estimate a set of parameters (or) to obtain an optimized model (or) which model yields the best prediction for future observations, so he has to have a knowledge on selection technique. Many statisticians have developed a different types of tests for selecting the models under two categories (i) Testing of Nested Regression Models and (ii) Tests of Non-Nested Regression Models.

If a model can be derived as a special case of the other then it is said to be nested model within the other model. Two models are said to be non-rested, if one cannot be derived a model as a special case of the other.

Consider the following two competing separate or non-nested linear regression models.

$$H_1 : Y = X_1\beta_1 + \in_1 \tag{6.1}$$

$$H_2 : Y = X_2\beta_2 + \in_2 \tag{6.2}$$

Where $Y, \in_1, \in_2$ are (nx1) vectors

X_1 and X_2 are (nxk_1) and (nxk_2) matrices with ranks K_1 and K_2 respectively. β_1 and β_2 are (k_1x1) and (k_2x1) vectors of unknown parameters.

These models (6.1) and (6.2) are non-nested models because the explanatory variables under one linear model are not a subset of the other linear model even though X_1 and X_2 may have some common variables.

Generally, it is assumed that Regressors in the right hand said of the model X_1 and X_2 are contemporaneously uncorrelated with the errors $\in_1$ and $\in_2$ of the true model. But, frequently, some of the columns of X_1 and X_2 matrices may be correlated with $\in_1$, the error terms of the true model. Thus, the OLS estimation is inappropriate. However, there is assumed exist a matrix of instruments, T, with the usual properties, so that two stage least squares estimation may be feasible. It may be explicitly assumed that both competing hypotheses specify the same matrix of instruments.

The two state least squares estimates of H_0 and H_1 and may be obtained by the OLS estimation of the models with the instrumental variables as,

$$Y = Q_T X_1\beta_1 + \in_1 \tag{6.3}$$

$$Y = Q_T X_2\beta_2 + \in_2 \tag{6.4}$$

Where $Q_T = T\left(T^1T\right)^{-1}T^1$

The two stage least square estimates of β_1 and β_2 are given by

$$\tilde{\beta}_1 = \left(X_1^1 Q_T X_1\right)^{-1}\left(X_1^1 Q_T Y\right)$$
(6.5)

and

$$\tilde{\beta}_2 = \left(X_2^1 Q_T X_2\right)^{-1}\left(X_2^1 Q_T Y\right)$$
(6.6)

Consider the artificial nested model, which is a linear combination of H_1 and H_2 as

$$H_3 : Y = (1-\alpha)X_1\beta_1 + \alpha X_2\beta_2 + \in$$
(6.7)

Where α is the unknown scalar

Replacing β_2 with its two stage least squares estimator $\tilde{\beta}_2$ and write the equation (7) as

$$Y = X_1\beta_1^* + \alpha X_2\tilde{\beta}_2 + \in$$
(6.8)

Where $\beta_1^* = (1-\alpha)\beta_1$

Now the two stage least squares estimate of (8) can be obtained by applying the OLS estimation to the model.

$$Y = Q_T X\beta_1^* + \alpha Q_{X_2T} Y + \in^*$$
(6.9)

where $Q_{X_2T} = Q_T X_2\left(X_2^1 Q_T X_2\right)^{-1} X_2^1 Q_T$

Multiplying both sides of (9) by

$$M_{X_1T} = \left[I - Q_T X_1\left(X_1^1 Q_T X_1\right)^{-1} X_1^1 Q_T\right]$$

gives

$$M_{X_1T}Y = \alpha M_{X_1T} Q_{X_2T} Y + \in^{**}$$
(6.10)

(or)

$$Y^* = \alpha Z + c^{**}$$
(6.11)

Where $Y^* = M_{X_1T}Y$, $Z = M_{X_1T} Q_{X_2T} Y$

To test for the significance of α from zero, the proposed t-test statistic is given by

$$t = \frac{Y^1 Q_{X_2T} M_{X_1T} Y}{\hat{\sigma}\sqrt{Y^1 Q_{X_2T} M_{X_1T} Q_{X_2T}}}$$
(6.12)

Where $\hat{\sigma}$ is the usual two stage least squares estimated standard error from equation (10). By using standard results, it can be shown that the estimate of α and its standard error from equation (11) are identical to those from (9), excepting for degrees of freedom corrections.

Under certain regularity conditions it can be shown that the test statistic given in (12) asymptotically follows N(0, 1) under H_1.

Now the above test procedure can be further modified as follows:

* Regress Y on X_1 and get the two stage least squares estimate of Y as $\tilde{Y}_1 = X_1\tilde{\beta}_1$

Where $\tilde{\beta}_1 = \left(X_1^1 Q_T X_1\right)^{-1}\left(X_1^1 Q_T Y\right)$

* Regress $\tilde{Y}_1$ on X_2 and get the two stage least squares estimate of Y as

$$\tilde{Y}_2 = X_2\left[\left(X_2^1 Q_T X_2\right)^{-1}\left(X_2^1 Q_T \tilde{Y}_1\right)\right]$$

* Regress Y on X_1 and $\tilde{Y}_2$ and test for the significance of coefficient of $\tilde{Y}_2$ from zero.

It is the t-statistic for the coefficient of $\tilde{Y}_2$. However, this t-statistic asymptotically follows N(0, 1).

One can test again by reversing the role of the hypotheses to know the asymmetry of H_1 and H_2

Under non –nested hypotheses testing, one may find the following four situations.

$\alpha = 1$

	Not Rejected	Rejected
Not Rejected	Both H_1 and H_2 are not rejected	H_1 rejected H_2 not rejected
Rejected	H_1 is not rejected H_2 is rejected	Both H_1 and H_2 are rejected

Both H_1 and H_2 are not $\Rightarrow$ The data are not rich

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rejected		enough to discriminate between the two hypotheses.
Both H_1 and H_2 are rejected	$\Rightarrow$	Neither model is useful in explaining the variation is Y.
Either of H_1 and H_2 is rejected	$\Rightarrow$	The non-rejected hypothesis may still be brought down by another challenger hypothesis.

VII. CONCLUSIONS:

In the present study an attempt has been made by describing various advanced tools for mathematical and stochastic modeling. Several Criteria for model building have been presented in the study. In this process of choosing models, statisticians have developed a variety of diagnostic tests. These tests have been classified into two categories namely (i).Tests of Nested Regression Models (ii).Tests of Non-Nested Regression Models.

A criterion for selecting non-nested regression model has been presented by using two stage least squares estimators. This criterion is conceptually much similar than most of the existing selection criteria.

. At present, there is a need to highlight some recant developments in Modeling from Mathematical and Stochastic point of view by Mathematicians and Statisticians.

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